1. For \( 0 \leq t \leq 6 \), a particle is moving along the \( x \)-axis. The particle’s position, \( x(t) \), is not explicitly given. The velocity of the particle is given by \( v(t) = 2\sin(\sqrt{t}/4) + 1 \). The acceleration of the particle is given by 
\[
a(t) = \frac{1}{2} e^{t/4} \cos(\sqrt{t}/4) + 1 \quad \text{and} \quad x(0) = 2.
\]
(a) Is the speed of the particle increasing or decreasing at time \( t = 5.5 \)? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period \( 0 \leq t \leq 6 \).

(c) Find the total distance traveled by the particle from time \( t = 0 \) to \( t = 6 \).

(d) For \( 0 \leq t \leq 6 \), the particle changes direction exactly once. Find the position of the particle at that time.

<table>
<thead>
<tr>
<th>( t ) (minutes)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H(t) ) (degrees Celsius)</td>
<td>66</td>
<td>60</td>
<td>52</td>
<td>44</td>
<td>43</td>
</tr>
</tbody>
</table>

2. As a pot of tea cools, the temperature of the tea is modeled by a differentiable function \( H \) for \( 0 \leq t \leq 10 \), where time \( t \) is measured in minutes and temperature \( H(t) \) is measured in degrees Celsius. Values of \( H(t) \) at selected values of time \( t \) are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time \( t = 3.5 \). Show the computations that lead to your answer.

(b) Using correct units, explain the meaning \( \frac{1}{10} \int_0^{10} H(t) \, dt \) in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate \( \frac{1}{10} \int_0^{10} H(t) \, dt \).

(c) Evaluate \( \int_0^{10} H'(t) \, dt \). Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time \( t = 0 \), biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time \( t \) is modeled by a differentiable function \( B \) for which it is known that 
\[
B'(t) = -13.84e^{-0.173t}.
\]
Using the given models, at time \( t = 10 \), how much cooler are the biscuits than the tea?
3. Let $R$ be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

(a) Write an equation for the line tangent to the graph of $f$ at $x = \frac{1}{2}$.

(b) Find the area of $R$.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when $R$ is rotated about the horizontal line $y = 1$.

4. The continuous function $f$ is defined on the interval $-4 \leq t \leq 3$. The graph of $f$ consists of two quarter circles and one line segment, as shown in the figure below. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the $x$-coordinate of the point at which $g$ has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.

(c) Find all values of $x$ on the interval $-4 < x < 3$ for which the graph of $g$ has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of $f$ on the interval $-4 \leq x \leq 3$. There is no point $c$, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.
5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function $W$ models the total amount of solid waste stored at the landfill. Planners estimate that $W$ will satisfy the differential equation 
\[
\frac{dW}{dt} = \frac{1}{25} (W - 300)
\] for the next 20 years. $W$ is measured in tons, and $t$ is measured in years from the start of 2010.

(a) Use the line tangent to the graph of $W$ at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of $W$. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25} (W - 300)$ with initial condition $W(0) = 1400$.

6. Let $f$ be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \leq 0 \\ e^{-4x} & \text{for } x > 0. \end{cases}$

(a) Show that $f$ is continuous at $x = 0$.

(b) For $x \neq 0$, express $f'(x)$ as a piecewise-defined function. Find the value of $x$ for which $f'(x) = -3$.

(c) Find the average value of $f$ on the interval $[-1, 1]$. 
1. The temperature of water in a tub at time $t$ is modeled by a strictly increasing, twice-differentiable function $W$, where $W(t)$ is measured in degrees Fahrenheit and $t$ is measured in minutes. At time $t = 0$, the temperature of the water is $55^\circ F$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times $t$ for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate \[
\int_0^{20} W'(t) \, dt .
\] Using correct units, interpret the meaning of $\int_0^{20} W'(t) \, dt$ in the context of this problem.

(c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) \, dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) \, dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \leq t \leq 25$, the function $W$ that models the water temperature has first derivative given by $W'(t) = 0.4 \sqrt{t} \cos(0.06t)$. Based on this model, what is the temperature of the water at time $t = 25$?

2. Let $R$ be the region in the first quadrant bounded by the $x$-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure below.

(a) Find the area of $R$.

(b) Region $R$ is the base of a solid. For the solid, each cross section perpendicular to the $x$-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides $R$ into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of $k$. 

\[
\text{\begin{tabular}{|c|c|c|c|c|c|} \hline \text{ } & \text{0} & \text{4} & \text{9} & \text{15} & \text{20} \\ \hline \text{W}(t) \text{ (degrees Fahrenheit)} & \text{55.0} & \text{57.1} & \text{61.8} & \text{67.9} & \text{71.0} \\ \hline \end{tabular}}
\]
3. Let $f$ be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let $g$ be the function given by $g(x) = \int_{-1}^{x} f(t) \, dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the $x$-coordinate of each point at which the graph of $g$ has a horizontal tangent line. For each of these points, determine whether $g$ has a relative minimum, relative maximum, or neither a minimum or a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of $x$ for which the graph of $g$ has a point of inflection. Explain your reasoning.

4. The function $f$ is defined by $f(x) = \sqrt{25-x^2}$ for $-5 \leq x \leq 5$.

(a) Find $f''(x)$.

(b) Write an equation for the line tangent to the graph of $f$ at $x = -3$.

(c) Let $g$ be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

Is $g$ continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_{0}^{5} x\sqrt{25-x^2} \, dx$. 

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time \( t = 0 \), when the bird is first weighed, its weight is 20 grams. If \( B(t) \) is the weight of the bird, in grams, at time \( t \) days after it is first weighed, then \( \frac{dB}{dt} = \frac{1}{5}(100 - B) \).

Let \( y = B(t) \) be the solution to the differential equation above with initial condition \( B(0) = 20 \).

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your answer.

(b) Find \( \frac{d^2B}{dt^2} \) in terms of \( B \). Use \( \frac{d^2B}{dt^2} \) to explain why the graph of \( B \) cannot resemble the following graph.

(c) Use separation of variables to find \( y = B(t) \), the particular solution to the differential equation with initial condition \( B(0) = 20 \).

6. For \( 0 \leq t \leq 12 \), a particle moves along the \( x \)-axis. The velocity of the particle at time \( t \) is given by \( v(t) = \cos\left(\frac{\pi}{6}t\right) \). The particle is at position \( x = -2 \) at time \( t = 0 \).

(a) For \( 0 \leq t \leq 12 \), when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time \( t = 0 \) to time \( t = 6 \).

(c) Find the acceleration of the particle at time \( t \). Is the speed of the particle increasing, decreasing, or neither at time \( t = 4 \)? Explain your reasoning.

(d) Find the position of the particle at time \( t = 4 \).
1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by \( G(t) = 90 + 45 \cos \left( \frac{t^2}{18} \right) \), where \( t \) is measured in hours and \( 0 \leq t \leq 8 \). At the beginning of the workday \((t = 0)\), the plant has 500 tons of unprocessed gravel. During the hours of operation, \( 0 \leq t \leq 8 \), the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find \( G'(5) \). Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time \( t = 5 \) hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

2. A particle moves along a straight line. For \( 0 \leq t \leq 5 \), the velocity of the particle is given by \( v(t) = -2 + \left( t^2 + 3t \right)^{6/5} - t^3 \), and the position of the particle is given by \( s(t) \). It is known that \( s(0) = 10 \).

(a) Find all values of \( t \) in the interval \( 2 \leq t \leq 4 \) for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position \( s(t) \). Use this expression to find the position of the particle at time \( t = 5 \).

(c) Find all times \( t \) in the interval \( 0 \leq t \leq 5 \) at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time \( t = 4 \)? Give a reason for your answer.
3. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time $t$, $0 \leq t \leq 6$, is given by a differentiable function $C$, where $t$ is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table below.

<table>
<thead>
<tr>
<th>$t$ (minutes)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(t)$ (ounces)</td>
<td>0.0</td>
<td>5.3</td>
<td>8.8</td>
<td>11.2</td>
<td>12.8</td>
<td>13.8</td>
<td>14.5</td>
</tr>
</tbody>
</table>

(a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

(b) Is there a time $t$, $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.

(c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^6 C(t) \, dt$. Using correct units, explain the meaning of $\int_0^6 C(t) \, dt$ in the context of the problem.

(d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

4. Consider the differential equation $\frac{dy}{dx} = e^y \left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1,0)$.

(a) Write an equation for the line tangent to the graph of $f$ at the point $(1,0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1,0)$.
5. The figure below shows the graph of $f'$, the derivative of a twice-differentiable function $f$, on the closed interval $0 \leq x \leq 8$. The graph of $f'$ has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of $f'$ and the x-axis are labeled in the figure. The function $f$ is defined for all real numbers and satisfies $f(8) = 4$.

![Graph of f'](image)

(a) Find all values of $x$ on the open interval $0 < x < 8$ for which the function $f$ has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of $f$ on the closed interval $0 \leq x \leq 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of $f$ both concave down and increasing? Explain your reasoning.

(d) The function $g$ is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of $g$ at $x = 3$.

6. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let $R$ be the region bounded by the graphs of $f$ and $g$, as shown in the figure below.

![Graph of R](image)

(a) Find the area of $R$.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 4$.

(c) The region $R$ is the base of a solid. For this solid, each cross-section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.
1. Grass clippings are placed in a bin, where they decompose. For $0 \leq t \leq 30$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 6.687 (0.931)^t$, where $A(t)$ is measured in pounds and $t$ is measured in days.

(a) Find the average rate of change of $A(t)$ over the interval $0 \leq t \leq 30$. Indicate units of measure.

(b) Find the value of $A'(15)$. Using correct units, interpret the meaning of the value in the context of the problem.

(c) Find the time $t$ for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \leq t \leq 30$.

(d) For $t > 30$, $L(t)$, the linear approximation to $A$ at $t = 30$, is a better model for the amount of grass clippings remaining in the bin. Use $L(t)$ to predict the time at which there will be 0.5 pounds of grass clippings remaining in the bin. Show the work that leads to your answer.

2. Let $R$ be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure below.

![Graph of f(x) and horizontal line y=4](image)

(a) Find the volume of the solid generated when $R$ is rotated about the horizontal line $y = -2$.

(b) Region $R$ is the base of a solid. For this solid, each cross sections perpendicular to the $x$-axis is an isosceles right triangle with a leg in $R$. Find the volume of the solid.

(c) The vertical line $x = k$ divides $R$ into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value of $k$. 
3. The function \( f \) is defined on the closed interval \([-5, 4]\). The graph of \( f \) consists of three line segments and is shown in the figure below. Let \( g \) be the function defined by \( g(x) = \int_{-3}^{x} f(t) \, dt \).

![Graph of f](image)

(a) Find \( g(3) \).

(b) On what open intervals contained in \(-5 < x < 4\) is the graph of \( g \) both increasing and concave down?

(c) The function \( h \) is defined by \( h(x) = \frac{g(x)}{5x} \). Find \( h'(3) \).

(d) The function \( p \) is defined by \( p(x) = f(x^2 - x) \). Find the slope of the line tangent to the graph of \( p \) at the point where \( x = -1 \).

4. Train A runs back and forth on an east-west section of railroad track. Train A’s velocity, measured in meters per minute, is given by a differentiable function \( v_A(t) \), where time \( t \) is measured in minutes. Selected values for \( v_A(t) \) are given in the table below.

<table>
<thead>
<tr>
<th>( t ) (minutes)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_A(t) ) (meters/minute)</td>
<td>0</td>
<td>100</td>
<td>40</td>
<td>-120</td>
<td>-150</td>
</tr>
</tbody>
</table>

(a) Find the average acceleration of train A over the interval \( 2 \leq t \leq 8 \).

(b) Do the data in the table support the conclusion that train A’s velocity is -100 meters per minute at some time \( t \) with \( 5 < t < 8 \)? Give a reason for your answer.

(c) At time \( t = 2 \) , train A’s position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A, in meters from the Origin Station, at time \( t = 12 \). Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time.

(d) A second train, train B, travels north from the Origin Station. At time \( t \) the velocity of train B is given by \( v_B(t) = -5t^2 + 60t + 25 \), and at time \( t = 2 \) the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time \( t = 2 \),
5. The twice-differentiable functions $f$ and $g$ are defined for all real numbers $x$. Value of $f$, $f'$, $g$, and $g'$ for various values of $x$ are given in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-2 &lt; x &lt; -1$</th>
<th>$-1$</th>
<th>$-1 &lt; x &lt; 1$</th>
<th>$1$</th>
<th>$1 &lt; x &lt; 3$</th>
<th>$3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>12</td>
<td>Positive</td>
<td>8</td>
<td>Positive</td>
<td>2</td>
<td>Positive</td>
<td>7</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>-5</td>
<td>Negative</td>
<td>0</td>
<td>Negative</td>
<td>0</td>
<td>Positive</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>$g(x)$</td>
<td>-1</td>
<td>Negative</td>
<td>0</td>
<td>Positive</td>
<td>3</td>
<td>Positive</td>
<td>1</td>
</tr>
<tr>
<td>$g'(x)$</td>
<td>2</td>
<td>Positive</td>
<td>$\frac{3}{2}$</td>
<td>Positive</td>
<td>0</td>
<td>Negative</td>
<td>-2</td>
</tr>
</tbody>
</table>

(a) Find the $x$-coordinate of each relative minimum of $f$ on the interval $[-2,3]$. Justify your answers.

(b) Explain why there must be a value $c$, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function $h$ is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x)) g'(x) \, dx$.

6. Consider the differential equation $\frac{dy}{dx} = (3 - y) \cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function $f$ is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0,1)$.

(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0,1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$. 