/20 if posted + then do at home, any above 20 is ½ pt (ex. 31 = 25.5)

/15 if done in class, ½ pt over 15 (ex: 31 = 23pts)

AP Calculus
Review #1 and #2

name ____

Calculators are not allowed. You have 45 minutes to complete questions 1-28.

1. If $y=(2x^2+1)^4$, then $\frac{dy}{dx}=$

Chain Rule $4(2x^2+1)^3\cdot 4x$

(A) $16x^3$ (B) $4(2x^2+1)^3$ (C) $4x(2x^2+1)^3$

(D) $16(2x^2+1)^3$ (E) $16x(2x^2+1)^3$

---

2. Find $\int x\sqrt{x^2+1}\,dx$.

$u=x^2+1$
$du=2x\,dx$
$dx=du/2x$
$\int x\,u^{1/2}\,du/2x=\frac{1}{2}\int u^{1/2}\,du$

(A) $\frac{x}{\sqrt{x^2+1}}+C$ (B) $\frac{3}{4}(x^2+1)^{\frac{3}{2}}+C$ (C) $\frac{1}{3}(x^2+1)^{\frac{3}{2}}+C$

$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$

(D) $\frac{2}{3}(x^2+1)^{\frac{3}{2}}+C$ (E) $\frac{1}{3}x^2(x^2+1)^{\frac{3}{2}}+C$

---

3. If $\frac{dy}{dx}=2xy$, then $\frac{d^2y}{dx^2}=$

$\frac{2x}{2}\quad\frac{y}{dy/dx}=2x\,dy/dx+2y=2x(2xy)+2y$
$=4x^2y+2y$

(A) $2y$ (B) $2x+2y$ (C) $2x+4y$

(D) $2x^2y+2y$ (E) $4x^2y+2y$

---

4. The graph of $y=2x^3+24x-18$ is

$y'=6x^2+24$ which is always positive

(A) increasing for all x
(B) decreasing for all x
(C) only increasing for all x such that $|x|>2$
(D) only increasing for all x such that $|x|<2$
(E) only decreasing for all x such that $x<-2$

---

5. If $f=\begin{cases}x^2+2, & x\le 1\\ 2x+1, & x>1\end{cases}$, then $f'(1)$ is

$f'=\begin{cases}2x\\ 2\end{cases}$

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 3 (E) nonexistent

6. What is the maximum value of the derivative of $f(x) = 3x^2 - x^3$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$f' = 6x - 3x^2$

$f'' = 6 - 6x$, $x = 1$, $f'(1) = 3$

---

7. Let f be a differentiable function for all x. Which of the following must be true?

I. $\frac{d}{dx}\int_0^3 f(x)\,dx = f(x)$ False

$\int_0^3 f(x)\,dx =$ a constant, so $d/dx = 0$

II. $\int_3^x f'(x)\,dx = f(x)$ False

$f(x)\Big|_3^x = f(x) - f(3)$, not $f(x)$

III. $\frac{d}{dx}\int_3^x f(x)\,dx = f(x)$

Area Acc, 2^{nd} Fund. Thm, True

(A) ~~II only~~
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

---

8. If $\sin(xy) = x^2$, then $\frac{dy}{dx} =$

$\cos xy \cdot (y + x\,dy/dx) = 2x$, $y + x\frac{dy}{dx} = \frac{2x}{\cos xy}$

(A) $2x\sec(xy)$
(B) $\frac{\sec(xy)}{x^2}$
(C) $2x\sec(xy) - y$

$x\,\frac{dy}{dx} = \frac{2x}{\cos xy} - y$

(D) $\frac{2x\sec(xy)}{y}$
(E) $\frac{2x\sec(xy) - y}{x}$

$\frac{dy}{dx} = \frac{2x}{x\cos xy} - \frac{y}{x}$

---

9. A particle moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 3t^3 - 18t^2 + 24t$. At which time t is its average velocity zero?

Initial position is 0, when is object at 0?

(A) Never
(B) 0 only
(C) 2 only
(D) ~~2 and 4 only~~
(E) 0, 2, and 4

$x(t) = 0 = 3t(t^2 - 6t + 8)$

0, 4, 2

10. How many points of inflection does the graph of $y = 2x^6 + 9x^5 + 10x^4 - x + 2$ have?

(A) None
(B) One
(C) Two
(D) Three
(E) Four

$y' = 12x^5 + 45x^4 + 40x^3 - 1$

$y'' = 60x^4 + 180x^3 + 120x =$

$60x^2(x^2 + 3x + 2) = 60x^2(x+2)(x+1)$

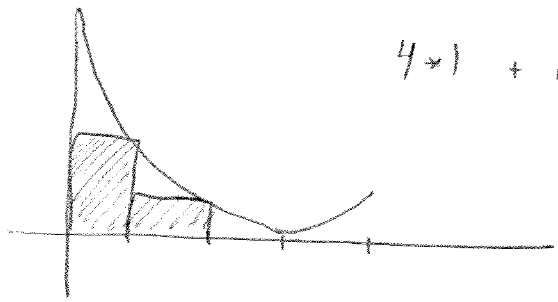
$+ \quad - \quad + \quad +$

$-2 \quad -1 \quad 0$

---

11. If $\int_0^4 (x^2 - 6x + 9)\,dx$ is approximated by 4 inscribed rectangles of equal width on the x-axis, then the approximation is

(A) 14
(B) 10
(C) 6
(D) 5
(E) 4



$4 \cdot 1 + 1 \cdot 1 = 5$

---

12. What is the 20$^{\text{th}}$ derivative of $y = \sin(2x)$

(A) $-2^{20}\sin(2x)$ (B) $2^{20}\sin(2x)$ (C) $-12^{19}\cos(2x)$

(D) $2^{20}\cos(2x)$ (E) $2^{21}\cos(2x)$

$y' = 2\cos 2x$, $y'' = -4\sin 2x$,

$y''' = -8\cos 2x$

$y'''' = 16\sin 2x$

$y^4 = 2^4 \sin 2x$

---

13. What is the equation of the line tangent to the graph of $f(x) = 7x - x^2$ at the point where $f'(x) = 3$?

(A) $y = 5x - 10$ (B) $y = 3x + 4$ (C) $y = 3x + 8$

(D) $y = 3x - 10$ (E) $y = 3x - 16$

$f'(x) = 7 - 2x = 3$, $x = 2$

$f(2) = 10$

$y - 10 = 3(x - 2)$

$y = 3x + 4$

---

14. If $f(x) = x\sqrt[3]{x}$, then $f'(x) =$

(A) $4x^3$ (B) $\frac{3}{7}x^{\frac{7}{3}}$ (C) $\frac{4}{3}x^{\frac{1}{3}}$ (D) $\frac{1}{3}x^{\frac{1}{3}}$ (E) $\frac{1}{3}x^{-\frac{2}{3}}$

$f(x) = x \cdot x^{\frac{1}{3}} = x^{4/3}$

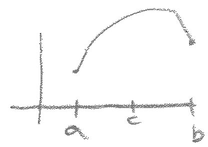
15. Suppose that $f(x)$ is a twice-differentiable function on the closed interval $[a, b]$. If there is a number c, $a < c < b$, for which $f'(c) = 0$, which of the following must be true?

I. $f(a) = f(b)$ NO
II. f has a relative extremum at $x = c$. NO
III. f has a point of inflection at $x = c$. NO

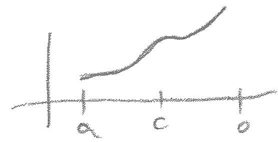
all could be, but don't have to be

(A) None
~~(B) I only~~
(C) II only
(D) I and II
(E) II and III

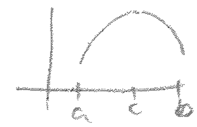
I.



II.



III



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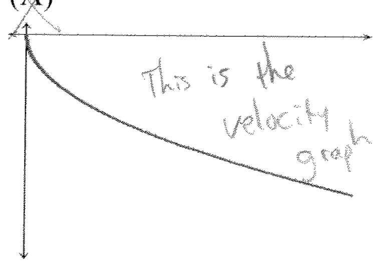
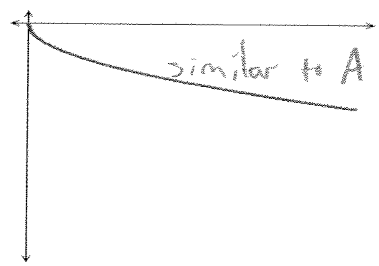
16. A sky diver has a negative velocity while falling from an airplane. Before the sky diver opens the parachute, her velocity decreases quickly and then levels off due to resistance. Which graph approximates the acceleration of the sky diver?

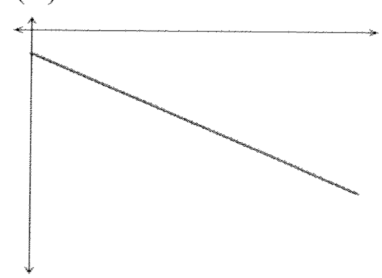
~~(A)~~



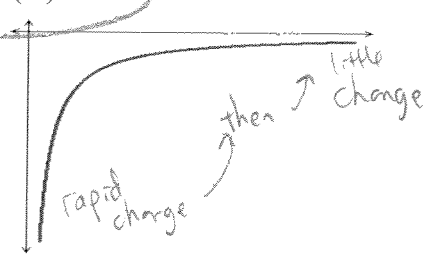
(B)



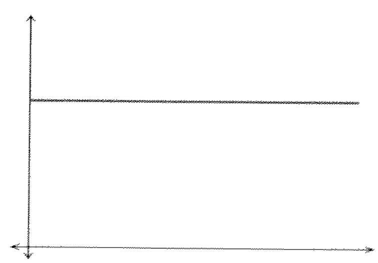
(C)



(D)



(E)



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17. If $k > 0$ and $\int_k^6 \frac{dx}{x+2} = \ln k$, then $k =$

~~(A) 1~~
(B) 2
(C) 3
(D) 4
(E) 5

$$\ln|x+2|\Big|_K^6 = \ln K$$

$$\ln 8 - \ln|K+2| = \ln K$$

$$\ln \frac{8}{K+2} = \ln K, \quad \frac{8}{K+2} = K$$

18. Find $\int_{e}^{e^2} \frac{dx}{x \ln x}$.

(A) ln 2
(B) ½
(C) 1
(D) 2
(E) e

$u = \ln x$
$du = \frac{1}{x} dx$
$dx = x\, du$

$\int_{e}^{e^2} \frac{1}{u} du$, $\ln u = \ln(\ln x)\Big|_{e}^{e^2}$

$\ln(\ln e^2) - \ln(\ln e)$

$\ln 2 - 0$

---

19. Let $f(x)$ be a continuous and differentiable function on the interval $0 \le x \le 1$, and let $g(x) = f(3x)$. The table below gives values of $f'(x)$, the derivative of $f(x)$. What is the value of $g'(0.1)$?

x	0.1	0.2	0.3	0.4	0.5	0.6
$f'(x)$	1.01	1.041	1.096	1.179	1.298	1.486

(A) 1.010
(B) 1.096
(C) 1.486
(D) ~~3.030~~
(E) 3.288

$g'(x) = f'(3x) \cdot 3$
$g'(.1) = f'(.3) \cdot 3 = 1.096 * 3$

---

20. For what value of k will $\frac{8x+k}{x^2}$ have a relative maximum at $x = 4$?

(A) ~~-32~~
(B) -16
(C) 0
(D) 16
(E) 32

$\frac{x^2(8) - (8x+k)2x}{x^4} = \frac{8x^2 - 16x^2 - 2kx}{x^4}$

$= \frac{-8x-2k}{x^3}$, $-8x - 2k = 0$
$-32 - 2k = 0$

---

21. The $\lim_{h \to 0} \frac{2(x+h)^5 - 5(x+h)^3 - 2x^5 + 5x^3}{h}$ is

(A) 0
(B) $10x^3 - 15x$
(C) ~~$10x^4 + 15x^2$~~
(D) $10x^4 - 15x^2$
(E) $-10x^4 + 15x^2$

what is the slope (derivative) of $2x^5 - 5x^3$?

22. If $\int_2^8 f(x)\,dx = -10$ and $\int_2^4 f(x)\,dx = 6$, then $\int_8^4 f(x)\,dx =$

(A) -16
(B) -6
(C) -4
(D) 4
(E) 16

$\int_2^8 = \int_2^4 + \int_4^8$

$-10 = 6 + (-16)$, $\int_4^8 f(x)dx = -16$, $\int_8^4 = 16$

---

23. If the graph of $y = x^3 + ax^2 + bx - 8$ has a point of inflection at (2, 0), what is the value of b?

(A) 0
(B) 4
(C) 8
(D) 12
(E) Cannot be determined from given information.

$y' = 3x^2 + 2ax + b$

$y'' = 6x + 2a$, $12 + 2a = 0$

$a = -6$

$y = (2)^3 + (-6)(2)^2 + b(2) - 8 = 0$

---

24. If $f(x) = x^{-\frac{1}{3}}$, what is the derivative of the inverse of $f(x)$?

(A) $x^{\frac{1}{3}}$ (B) $-\frac{1}{3}x^{-\frac{4}{3}}$ (C) $\frac{1}{3}x^{-\frac{2}{3}}$

(D) $-3x^{-2}$ (E) $-3x^{-4}$

Inverse: $y = x^{-1/3}$

$x = y^{-1/3}$

$x^3 = y^{-1}$

$\frac{1}{x^3} = y = x^{-3}$,

$-3x^{-4}$

---

25. If f is a continuous function on the closed interval [a, b], which of the following statements are NOT necessarily true?

I. f has a minimum on [a, b]. yes (b/c closed)
II. f has a maximum on [a, b]. yes
III. $f'(c) = 0$ for some number c, $a < c < b$.

NOT necessarily

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I, II, and III

26. The volume of a cube is increasing at the rate of 20 cubic centimeters per second. How fast, in square centimeters per second, is the surface area of the cube increasing at the instant when each edge of the cube is 10 centimeters long? $V = s^3$, $SA = 6s^2$, find dSA/dt

(A) $\frac{4}{3}$ (B) 2 (C) 4 (D) 6 (E) 8

$20 = 3(10^2)\, ds/dt$, $ds/dt = 20/300$ $\leftarrow$ $\frac{dV}{dt} = 3s^2\, ds/dt$ $\frac{dSA}{dt} = 12s\, ds/dt$, $= 120\, ds/dt$ substit

---

27. If $\frac{dy}{dx} = \frac{x}{y}$ and $y(3) = 4$, then $\int y\,dy = \int x\,dx$, $\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

$y^2 = x^2 + C$

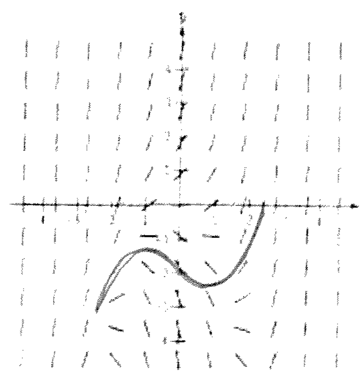
(A) $x^2 - y^2 = -7$ (B) $x^2 + y^2 = 7^2$ (C) $x^2 - y^2 = 7$

(D) $y^2 - x^2 = 5$ (E) $y^2 - x^2 = 7^2$

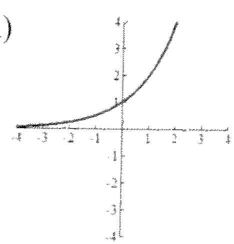
$16 = 9 + C$, $C = 7$

---

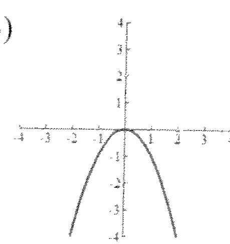
28. Which of the following graphs could be a particular solution of the differential equation whose slope field is below?



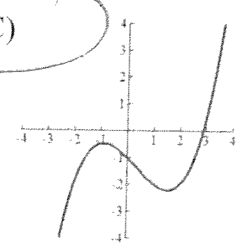
(A)



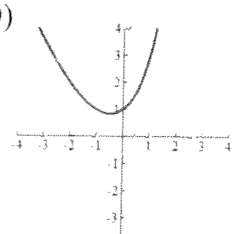
(B)



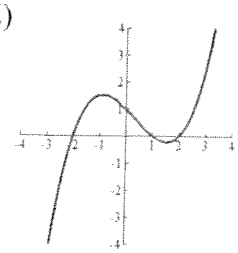
(C)



(D)

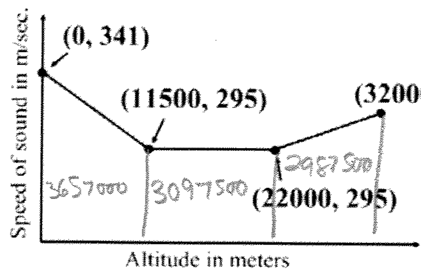


(E)



Calculators are allowed. You have 45 minutes to complete questions 29-43.

29.



Find 3 areas / 32000

In the earth's atmosphere the speed of sound is a function of the altitude. The figure above consisting of 3 line segments shows the speed of sound $s(a)$ in m/sec. as a function of altitude, a in meters. The graph is not drawn to scale. What is the average speed of sound in m/sec. on the interval [0, 32,000]?

(A) 295 (B) 303.9 (C) 304.4 (D) 306.8 (E) 312.8

---

30. Let f be a continuous function such that $\int_2^3 f(2x)\,dx = 8$. What is the value of $\int_4^6 f(x)\,dx$?

(A) 4
(B) 8
(C) 12
(D) 16
(E) 32

$u = 2x$ → new boundaries would be 6 + 4

$du = 2dx$

$dx = du/2$

$\frac{1}{2}\int_4^6 f(u)\,du = 8, \quad \int_4^6 = 16$

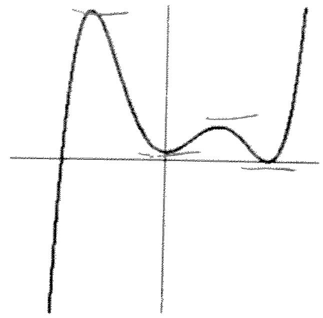
---

31. If $f(x) = 2x^3$, then the average rate of change of f on the interval [0, 2] is

(F) 4
(G) 8
(H) 12
(I) 16
(J) 24

$\frac{f(0) - f(2)}{0 - 2} \qquad \frac{0 - 16}{0 - 2} = 8$

32.



If the complete polynomial graph of $f(x)$ is given above, then the graph of $f'(x)$, the derivative of $f(x)$, will cross the x-axis in exactly how many points?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

---

33. At what point on the curve $x^3 - y^2 + x^2 = 0$ is the tangent line vertical?

$3x^2 - 2y\,dy/dx + 2x = 0$

(A) (0, 0) only (B) (-1, 0) only (C) $(1, \sqrt{2})$ only

(D) (-1, 0) and (0, 0) (E) The tangent line is never vertical.

bottom = 0

$\frac{dy}{dx} = \frac{-3x^2 - 2x}{-2y}$

$-2y = 0$, $\boxed{y = 0}$ plus into original + solve for x

---

34. A company manufactures x calculators weekly that can be sold for $75 - 0.01x$ dollars each. The cost of manufacturing x calculators is given by $1850 - 28x - x^2 + 0.001x^3$. The number of calculators the company should manufacture weekly in order to maximize its weekly profit is

(A) 611
(B) ~~652~~
(C) 683
(D) 749
(E) 754

Profit = Revene - cost

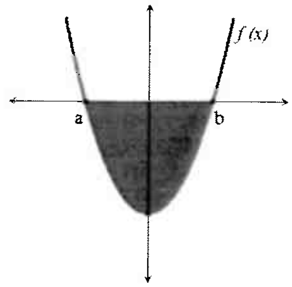
$P = x(75 - .01x) - (1850 - 28x - x^2 + .001x^3)$

CALCS $/CALC - $

\$ - \$

Put into y_1
find max

35.



If f is the continuous function shown in the figure above, then the area of the shaded region is

(A) $\int_a^b f(x)\,dx$ −area

(B) $\int_b^a f(x)\,dx$ +area

(C) $\int_b^{-a} f(x)\,dx$

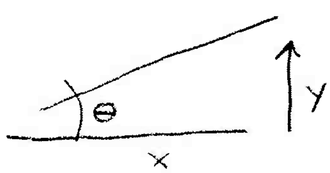
(D) $\int_a^{-b} f(x)\,dx$

(E) $\int_{-b}^{-a} f(x)\,dx$

---

36. A missile rises vertically from a point on the ground 75,000 feet from a radar station. If the missile is rising at the rate of 16,500 feet per minute at the instant when it is 38,000 feet high, what is the rate of change, in radians per minute, of the missile's angle of elevation from the radar station at this instant?

(A) 0.175
~~(B) 0.219~~
(C) 0.227
(D) 0.469
(E) 0.507



$\tan\theta = y/75000$ $\theta = \tan^{-1}\left(\frac{38000}{75000}\right)$

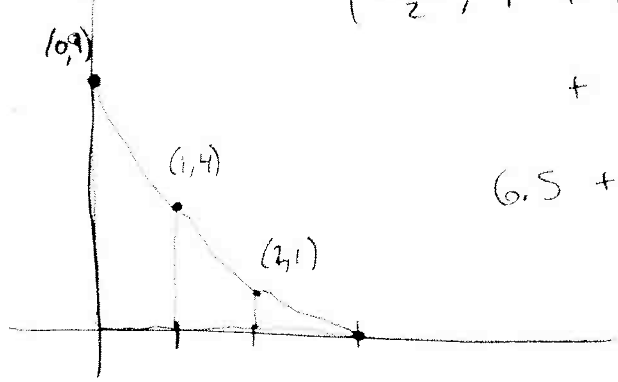
$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{75000}\, dy/dt$

$\frac{d\theta}{dt} = \frac{1}{75000}(16500)\cdot\cos^2\theta$

---

37. If three equal subdivisions of [0, 3] are used, what is the Trapezoidal Rule approximation of $\int_0^3 (x^2 - 6x + 9)\,dx$?

(A) 3
~~(B) 9~~
(C) 9.5
(D) 10
(E) 19



$\left(\frac{9+4}{2}\right)\cdot 1 + \left(\frac{4+1}{2}\right)\cdot 1 + \left(\frac{1+0}{2}\right)\cdot 1$

$6.5 + 2.5 + .5$

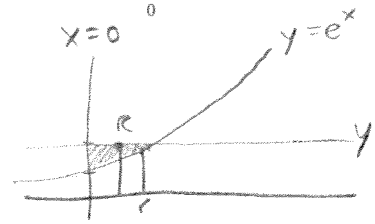
38. Let R be the region in the first quadrant enclosed by the lines $x = 0$ and $y = 2$ and the graph of $y = e^x$. The volume of the solid generated when R is revolved about the x-axis is given by

(A) $\pi\int_0^2 \left(4-e^{2x}\right)dx$ (B) $\pi\int_0^{\ln 2} \left(2-e^x\right)^2 dx$ (C) $2\pi\int_0^{\ln 2} x\left(2-e^x\right)dx$

(D) $\pi\int_0^{\ln 2} \left(4-e^{2x}\right)dx$ (E) $2\pi\int_0^2 x\left(2-e^x\right)dx$

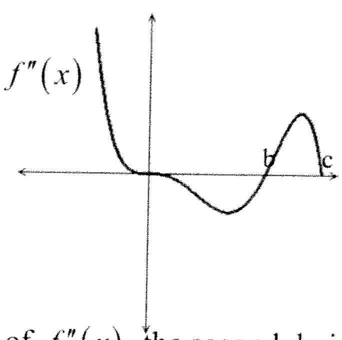
$R = 2 - 0$

$r = e^x - 0$



$\pi \int 2^2 - (e^x)^2 dx$

39.



The figure above shows the graph of $f''(x)$, the second derivative of a function $f(x)$. The function $f(x)$ is continuous for all x. Which of the following statements about f are true?

a. f is concave up for $x < 0$ and $b < x < c$. yes b/c $f'' > 0$
b. f has a relative minimum in the open interval $b < x < c$. ??
c. f has points of inflection at $x = 0$ and $x = b$. yes, f'' changes signs

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III only

40. Which statement is true for the function $f(x) = \ln(\tan x)$ on the open interval $\pi < x < \frac{5\pi}{4}$?

(A) $f(x)$ is increasing at an increasing rate. Look at graph on Calc
(B) $f(x)$ is increasing at a decreasing rate. Concave down
(C) $f(x)$ has an absolute maximum in the open interval.
(D) $f(x)$ has a point of inflection in the open interval.
(E) $f(x)$ has a point of symmetry in the open interval.

41. If $\lim_{x \to 2} \frac{f(x)}{x-2} = f'(2) = 0$, which of the following must be true?

I. $f(2) = 0$ yes

II. $f(x)$ is continuous at $x = 2$ yes b/c $f'(2)$ exists

III. $f(x)$ has a horizontal tangent line at $x = 2$ yes b/c $f'(2) = 0$

(A) I only
(B) II only
(C) I and II only
(D) ~~II and III only~~
(E) I, II, and III

$$\lim_{x \to 2} \frac{f(x)}{x-2} = f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x-2}$$

if =, then $f(2) = 0$

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42. How many extrema (maximum and minimum) does the function $f(x) = (x+2)^3 (x-5)^2$ have on the open interval $-3 < x < 6$?

(A) None
(B) ~~One~~
(C) Two
(D) Three
(E) Four

Graph

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43. If $\int_0^{1000} 8^x\,dx - \int_a^{1000} 8^x\,dx = 10.40$, then $a =$

(A) ~~1.4~~
(B) 1.5
(C) ~~1.6~~
(D) 1.7
(E) 1.8

$$\left.\frac{8^x}{\ln 8}\right|_0^{1000} = \frac{8^{1000}}{\ln 8} - \frac{1}{\ln 8}$$

$$10.40 = \left(\frac{8^{1000}}{\ln 8} - \frac{1}{\ln 8}\right) - \left(\frac{8^{1000}}{\ln 8} - \frac{8^a}{\ln 8}\right) = \frac{8^a}{\ln 8} - \frac{1}{\ln 8} = 10.40$$

put in y_1 put in y_2

find intersection

$$\left.\frac{8^x}{\ln 8}\right|_a^{1000} = \frac{8^{1000}}{\ln 8} - \frac{8^a}{\ln 8}$$